#16

Game theory (continued)

Game G = (players, strategies, outcomes, information)

Nash Equilibrium (N.E.):

Every player can only get worse if he/she changes his/her own strategy.

(Not every game has a N.E. state or there can be multiple N.E. states in a game.)

Consider a game with two players and each has two stragegies with complete information.

		P2	
		Strategy B1	Strategy B2
P1	Strategy A1	$c_{11}, \tilde{c_{11}}$	$c_{12}, \tilde{c_{12}}$
	Strategy A2	$c_{21}, \tilde{c_{21}}$	$c_{22}, \tilde{c_{22}}$

$c_{ij}, \tilde{c_{ij}}$: Payoff of P1 and P2 respectively for strategies A_i and B_j

Q: How to find all the Nash Equilibrium states for this game? -> discussion for 5 minutes

Try your method on the following games.

		B	
		Keep silence (cooperate)	Betray (defect)
A	Keep silence (cooperate)	A: 1 year B: 1year	A: 3 years B: free
	Betray (defect)	A: free B: 3 years	A: 2 years B: 2 years

Prisoner's Dilemma (M. Flood and M. Dreshev, A.W. Tucker, 1950)

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		米国 (USA)	
		軍縮 arms cut	軍拡 expansion
ソ連 (USSP)	軍縮 arms cut	3, 3	-1, 4
	軍拡 expansion	4, -1	-2, -2

「大国日本の世渡り学ー国際摩擦を考える」
高坂正堯，PHP文庫, 1990, p162

		Big pig	
		Press button	Wait for food
Little pig	Press button	1, 5	-1, 9
	Wait for food	4, 4	0, 0

Algorithm for finding all the N.E. states:

		P2	
		Strategy B1	Strategy B2
P1	Strategy A1	$c_{11}, \tilde{c_{11}}$	$c_{12}, \tilde{c_{12}}$
	Strategy A2	$c_{21}, \tilde{c_{21}}$	$c_{22}, \tilde{c_{22}}$

for all columns j = 1, 2, ...; do

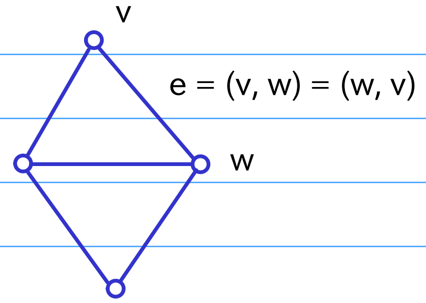
find the maximum entries c_{ij} among column j

if $\tilde{c_{ij}}$ is a maximum entry w.r.t row i, then (i, j) is an N. E.

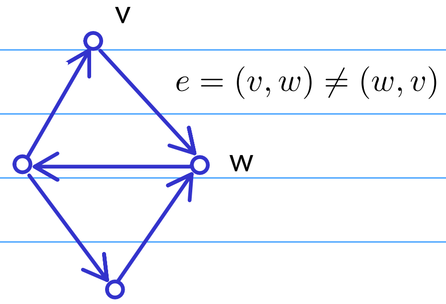
Mini Report #2

Graph

G = (V, E), where V is a set of n vertics and E is a set of m edges.



(undirected) graph



directed graph (or digraph)

Ex.: social network (friend network)

Ex.: road network, twitter network

Graph and Network:

We usually use graph to denote the structure and use network to denote a graph associated with weight or property on nodes or edges or both.

More examples:

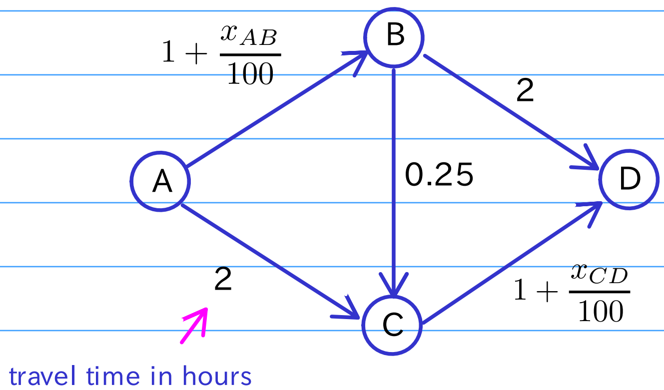
* Food web (who eats what in the nature)
* Partnership among companies
* World-Wide Web (WWW)
* Citation network
* Language network
* Metabolic and protein network
* ...

The network in Mini Report #1 (10 minutes)

#19

Network traffic

Example from Wikipedia -> Nash_equilibrium



* a network of 4 cities A, B, C, and D

* 100 cars from A to D

* Tavel time given in the figure, where x_{AB} and x_{CD} show the #cars.

What would be the distribution of cars?

Let us consider an N.E. state, i.e. a state no driver can benefit from changing route.

There are 3 routes in total:

R1: A -> B -> D with travel time 3 + x_{AB}/100

R2: A -> C -> D with time 3 + x_{CD}/100

R3: A -> B -> C -> D with time 2.25 + (x_{AB} + x_{CD})/100

In an N.E. state, we assume all routes have the same travel time.

=> 3 + x_{AB}/100 = 3 + x_{CD}/100 = 2.25 + (x_{AB} + x_{CD})/100

=> x_{AB} = x_{CD} = 75

=> travel time = 3.75 hours.

=> #cars using routes R1, R2, R3 are 25, 50, 25, respectively.

What would happen if we remove the B->C link?

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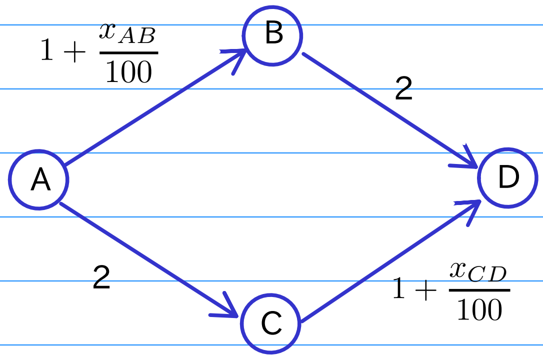
Network traffic (continued)

If we remove the B->C link, then there are only two routes.

R1: A -> B -> D with time 3 + x_{AB}/100

R2: A -> C -> D with time 3 + x_{CB}/100



In an N.E. state, we assume all routes have the same travel time.

=> 3 + x_{AB}/100 = 3 + x_{CD}/100, x_{AB} + x_{CD} = 100

=> x_{AB} = x_{CD} = 50 with travel time 3.5 hours, more efficient than before!

This example shows that sometimes adding a link may decrease the efficiency of a traffic system, or, in other words, removing a link may improve the efficiency.

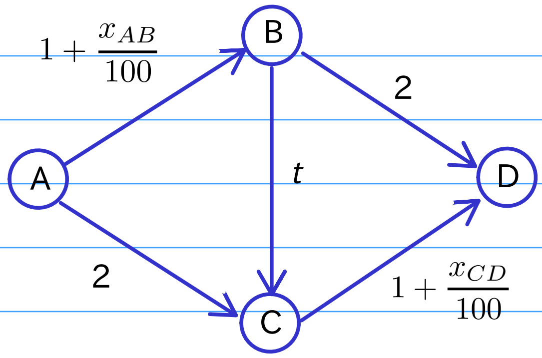
This is known as the Braess' Paradox named by its discoverer D. Braess (mathematician). It demonstrates that adding something may not always be the best choice, sometimes we need to remove (to forget) something for a better system.

We want to do something good (e.g., adding a fast shortcut to "improve" the traffic situation). But we need careful consideration. In fact, there are many bad examples in law, economics, politics, and other social systems. It is important for people working in social areas to learn some mathematics than those who work in scientific areas!

"[S]ocial sciences often take the lazy road of fitting raw data with a straight line or some fashionable format, unaware of the need to think and build models based on logic ...
I call for a major widening in social science methodology."
Taagepera R. Science walks on two legs, but social sciences try to hop on one.
International Political Science Review. 2018;39(1):145-159.

Exercise and discussion

Consider the N.E. situation for the above network with parameter t.



What can you observe by thinking about the N.E. for the above network? -> 15 minutes

Answer: the min travel time is 3.5 for t>=0.5, otherwise 4-t (faster BC => longer time).