

## Review of some fundamental Math.

1. Numbers: one of the most fundamental studies to learn the world

- \* when did you begin to count 1, 2, ..., 10?
- \* natural numbers:  $\mathbb{N} = \{1, 2, \dots\}$  (<- why "natural"?)
- \* integer numbers:  $\mathbb{Z} = \{0, +/-1, +/-2, \dots\}$  (<- why we need them?)
- \* rational numbers:  $\mathbb{Q}$  (<- why "rational" and why we need them?)
- \* real numbers:  $\mathbb{R}$  (<- why "real" and why we need them?)

$\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$

What are on the axis?



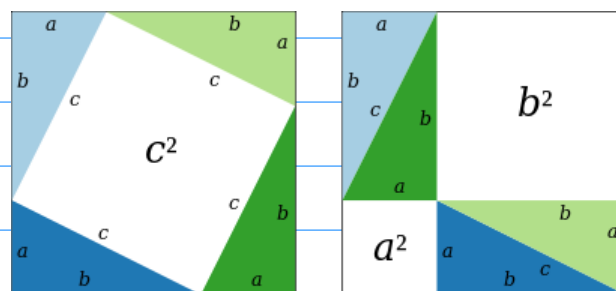
Irrational numbers:  $\mathbb{R} - \mathbb{Q}$ , i.e., a set of numbers that cannot be denoted by  $p/q$  where  $p, q \in \mathbb{Z}$  and  $q \neq 0$ .

Ex. Show  $\sqrt{2}$  is an irrational number.

Hint: proof by contradiction.

Remark: (a.b. BC 500)

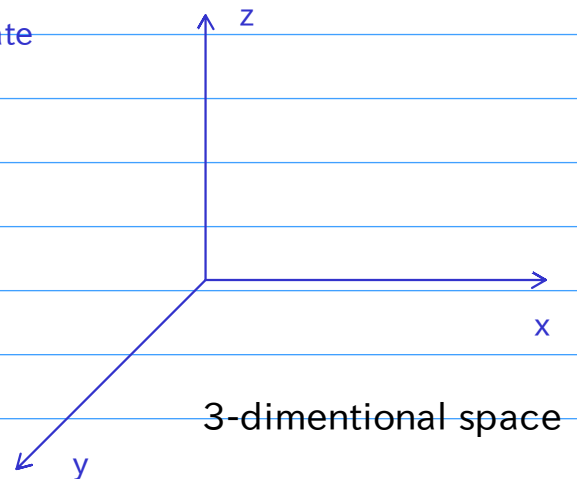
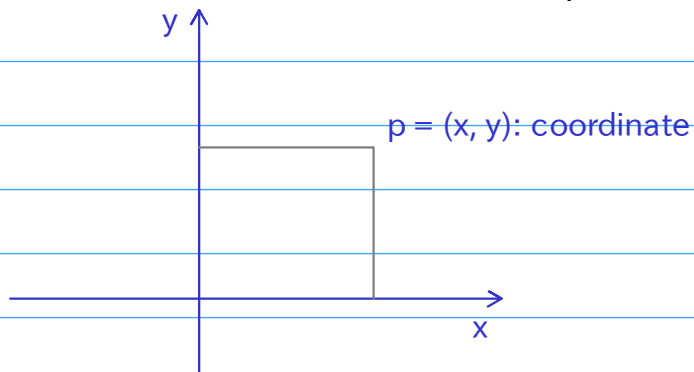
Pythagorean theorem, Pythagorean school and  $\sqrt{2}$ .



$$c^2 = a^2 + b^2$$

## Space

## 2-dimensional space

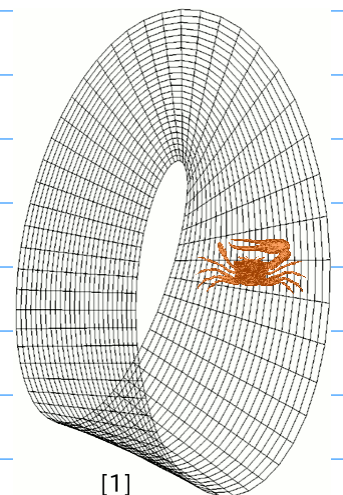
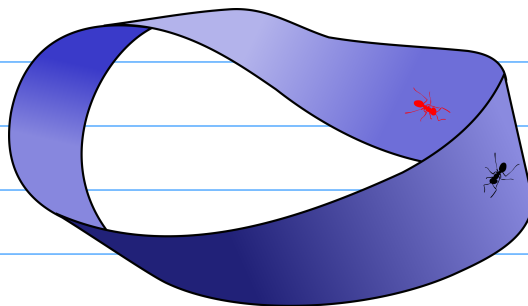


## n-dimensional space

$$x = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$$

Further reading: How many dimensions do we live in?

## Mobius strip: studied by Mobius, 1865



[1]

- \* What is the dimension of this strip?
- \* Application: printer's ribbon (USA patent), etc
- \* Can the ants/crab know the dimension of the space they live?
- \* String theory suggests more secret dimensions.



<https://www.capitalfm.com/artists/zayn/one-direction-left-band-five-years-post-quit/>

<https://time.com/3836129/stephen-hawking-one-direction-zayn-malik-multiple-universes/>



AMERICA'S CLIMATE REAL ESTATE BUBBLE

VIDEO



NEWSLETTE

After the lecture, one audience member asked, “What do you think is the cosmological effect of Zayn leaving One Direction and consequently breaking the hearts of millions of teenage girls across the world?”

And Hawking’s response was genius.

“Finally, a question about something important,” he said. “My advice to any heartbroken young girl is to pay close attention to the study of theoretical physics. Because one day there may well be proof of multiple universes.”

“It would not be beyond the realms of possibility that somewhere outside of our own universe lies another different universe — and in that universe, Zayn is still in One Direction.”

Hawking went on to say that, “this girl may like to know that in another possible universe, she and Zayn are happily married.”

## Operators on matrix/vector

Transpose:

$$A = (a_{ij}) \in \mathbb{R}^{m \times n} \Rightarrow A^T = (a_{ji}) \in \mathbb{R}^{n \times m}$$

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} \in \mathbb{R}^{n \times 1} \Rightarrow x^T = (x_1, x_2, \dots, x_n) \in \mathbb{R}^{1 \times n}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \in \mathbb{R}^{m \times n} \Rightarrow A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix} \in \mathbb{R}^{n \times m}$$

Calculations

$$+ - : A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{m \times n} \Rightarrow$$

$$A \pm B = \begin{pmatrix} a_{11} \pm b_{11} & \cdots & a_{1j} \pm b_{1j} & \cdots & a_{1n} \pm b_{1n} \\ \vdots & \ddots & \vdots & & \vdots \\ a_{i1} \pm b_{i1} & \cdots & a_{ij} \pm b_{ij} & \cdots & a_{in} \pm b_{in} \\ \vdots & & \vdots & \ddots & \vdots \\ a_{m1} \pm b_{m1} & \cdots & a_{mj} \pm b_{mj} & \cdots & a_{mn} \pm b_{mn} \end{pmatrix} \in \mathbb{R}^{m \times n}$$

$j$

$$A \in \mathbb{R}^{m \times p}, B \in \mathbb{R}^{p \times n} \Rightarrow$$

$$AB = (c_{ij}) \in \mathbb{R}^{m \times n}, \text{ where}$$

$$c_{ij} = \sum_{k=1}^p a_{ik} b_{kj}$$

Prisoner's Dilemma (M. Flood and M. Dreshev, A.W. Tucker, 1950)

		B	
		Keep silence (cooperate)	Betray (defect)
A	Keep silence (cooperate)	A: 1 year B: 1 year	A: 3 years B: free
	Betray (defect)	A: free B: 3 years	A: 2 years B: 2 years

What would be the most possible (rational) case? => A: ? and B: ?

This tells us that two purely rational individuals may not cooperate, even if they know cooperation is the best choice.

Ex. 大阪のデタラメ!市職員の5人以上の会食200件、千人以上参加の仰天「上司の強要、偽装工作」核心証言〈dot.〉

4/24(土) 12:30 配信 5780  

**AERA**dot.



松井大阪市長(左)と吉村大阪府知事  
(C)朝日新聞社

新型コロナウイルスの感染拡大が止まらず、緊急事態宣言が出た大阪。大阪市職員が3月1日から4月4日の間に5人以上での会食などを行っていたケースは200件以上、参加した職員は1000人を超えるなどデタラメな実態が明らかになった。

【独自】コロナ禍で「身を切る」全国知事の給与・ボーナスランキング 吉村知事は？

大阪市は3月1日、市内の飲食店に午後9時までの営業時間の短縮を求め、市民には「4人以下でのマスク会食」を呼びかけていた。しかし、3月末に大阪市の高齢施設課職員9人が飲食店で送別会を開催。別の部署でも5人で会食するなど合計で5人が新型コロナウイルス感染したことから、全職員を対象に調査していた。

大阪市の松井一郎市長は記者会見で「本当に申し訳ないと思う、皆さんに」と陳謝した。

大阪市の調査に対して、5人以上で飲食をしていたと回答した職員のAさんはこう話す。

「1週間ほど前に聞かれました。本当のことを話すか悩みましたが、参加者は複数いて、誰かが真実をしゃべるとウソつきになると思って正直に答えました」

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## Nash Equilibrium (N.E.)

All players can only get worse if he/she changes his/her own strategy.

Application: marketing, plea-bargain (司法取引, 认罪协商)

### Game:

\* player

\* strategy

\* outcome

\* information

Notice that not every game has a N.E. state or there can be multiple N.E. states in a game.

payoff matrix

		B	
		cooperate	defect
A	cooperate	R, R	T, S
	defect	T, S	P, P

Supposing a larger value is better in the above payoff matrix, under what condition would a Prisoner's Dilemma happen?

That is, both A and B, if they are rational, would choose to defect.

$$S < P < R < T$$

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## A game in real situation

「大国日本の世渡り学—国際摩擦を考える」

高坂正堯, PHP文庫, 1990, p162



<http://samurai-models.blog.jp/archives/24688978.html>

米ソ軍事競争の利得表  
(payoff matrix for USA/USSP's arms race)

		米国 (USA)	
		軍縮 arms cut	軍拡 expansion
ソ連 (USSP)	軍縮 arms cut	3, 3	-1, 4
	軍拡 expansion	4, -1	-2, -2

Q: Is there a Nash Equilibrium state in this game?

This example demonstrates that human can be irrational in real situations.

It is called a chicken game usually and military contractors benefits from it.

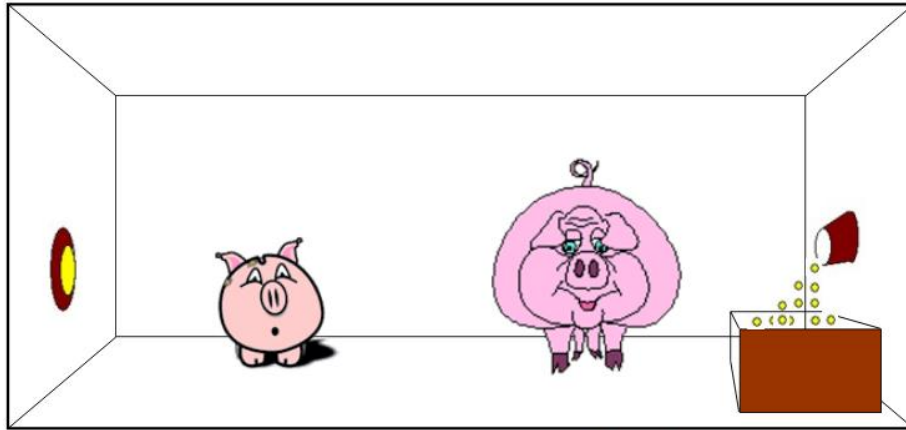
Other examples of chicken game include the trade wars, Cuban missile crisis, etc



A US Navy P-2H Neptune of VP-18 flying over a Soviet cargo ship with crated II-28s on deck during the Cuban Crisis.  
USN - Dictionary of American Naval Aviation Squadrons Volume 2: The History of VP, VPB, VP(H) and VP(AM) Squadrons [4],  
Public domain, <https://commons.wikimedia.org/w/index.php?curid=7328539>



### Another Example: Big & Little Pigs



Cost to press  
button = 2 units

When button is pressed,  
food given = 10 units

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<https://slideplayer.com/slide/4544128/>

		Big pig	
		Press button	Wait for food
Little pig	Press button	1, 5	-1, 9
	Wait for food	4, 4	0, 0

Q: Is there a Nash Equilibrium state in this game?

This example demonstrates Dominant Strategy, i.e., a strategy better than others.

Here, the dominant strategy for little pig is to wait, whereas for big pig to press. Why?

Application: R&D for developing new market (big company vs small company)