#22

Pareto efficiciency (also Pareto Optimality)

Nash Equilibrium (N.E.): a rational optimality

Any players can only get worse if he/she changes his/her own strategy.

Prisoner's Dilemma

		B: Keep silence (cooperate)	B: Betray (defect)
A	Keep silence (cooperate)	A: 1 year B: 1year	A: 3 years B: free
A	Betray (defect)	A: free B: 3 years	A: 2 years B: 2 years

P.E. → (Keep silence, Keep silence)

N.E. → (Betray, Betray)

Pareto Efficiency (P.E.): a (relative) global optimality

Impossible to make one better off without making at least one another worse off.

Two pigs' Game

		Big pig: Press button	Big pig: Wait for food
Little pig	Press button	1, 5	-1, 9
Little pig	Wait for food	4, 4	0, 0

These exampels demonstrate that a rational strategy may and may not be P.E..

We shall realize the difference and try to find a strategy satisfying more criteria.

#23

Life is not always rational

The chicken game demonstrates that human can be irrational in real situations.

Also called the Hawk-Dove Game

		Player B	
		Hawk	Dove
Player A	Hawk	-1000, -1000	1, -1
	Dove	-1, 1	0, 0

Chicken game: https://www.youtube.com/watch?v=u7hZ9jKrwvo

Chicken game can be found in politics, economics, biology and others.
It (Or, in general, irrational activities of life) can be explained as a result of the nature. More detail can be found from my lecture in the second semester about Information Wisdom Theory (in Japanese now).

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Volunteer's dilemma

N players need to decide if to make a small sacrifice from which all will benefit.

Payoff of a player i (i=1, 2, ..., N)

	at least one other people cooperate	all others defect
cooperate	0	-1
defect	1	-10

N = 2 => chicken game.

A real (but somewhat misleading) story: murder of Kitty Gerovese in 1964.
(See more detail on Wikipedia -> Murder_of_Kitty_Genovese)

New York Times reported that neighbors saw or heard murder but didn't call the police.
"37 Who Saw Murder Didn't Call the Police; Apathy at Stabbing of Queens Woman Shocks Inspector" - NYT, March 27, 1964

PS. The latest report by NYT in 2016 speaks that the number of witness was not clear and two of the neighbors did call the police.

As a summary, human are not always rational and in many real cases, the (most rational) N.E. strategies were not chosen.

#25

True bidding (incentive compatibility)

Four standard auction rules:

1. First-price, Sealed-bid

E.g., Goverment, Organization, etc

2. Second-price, Sealed-bid (Vickrey auction)

E.g., online auction (see later discussion)

3. Open ascending-bid (English auction)

Start with a low price and bidder with the highest price buys (common)

4. Open descending-bid (Dutch auction)

Start with a high price and bidder answers the first buys (for Dutch Tulip)

Second-price, Sealed-bid (SPSB)

* Can be back to at least 1893 and 1797.

* First academically described by William Vickrey in 1961.

↖ Novel Laureate (1996, Economics Sciences)

Theorem

In SPSB, bidding with true value is a dominant strategy for each bidder.

Proof

Let v_i and b_i be the true value and bidding value w.r.t bidder i.

Assume all b_i are distinct for simplicity.

The payoff p_i of bidder i is thus

$$p_i = \begin{cases} 0, & b_i < \max\{b_j\} \\ v_i - \max_{j \neq i}\{b_j\}, & b_i \geq \max\{b_j\} \end{cases}$$

(to next page)

(proof continued)

Compare the payoff of true bidding and overbidding.

cases	b_i = v_i	b_i > v_i
$v_i > \max_{j \neq i}\{b_j\}$	$v_i - \max_{j \neq i}\{b_j\} > 0$	$v_i - \max_{j \neq i}\{b_j\} > 0$
$v_i \leq \max_{j \neq i}\{b_j\}$ $b_i < \max_{j \neq i}\{b_j\}$	0	0
$v_i \leq \max_{j \neq i}\{b_j\}$ $b_i > \max_{j \neq i}\{b_j\}$	-	$v_i - \max_{j \neq i}\{b_j\} \leq 0$

Therefore true bidding is dominant than overbidding.

Next, compare the payoff of true bidding and underbidding.

Therefore true bidding is dominant than underbidding.

Finally we found true bidding is dominant. This is called "incentive compatibility".

Weakness of SPSB:

Not allow for price discovery; Vulnerable to bidder collusion; Vulnerable to multiple identities; Seller revenues; etc

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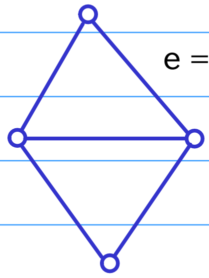
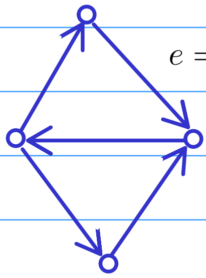
Graph

G = (V, E), where V is a set of n vertics and E is a set of m edges.

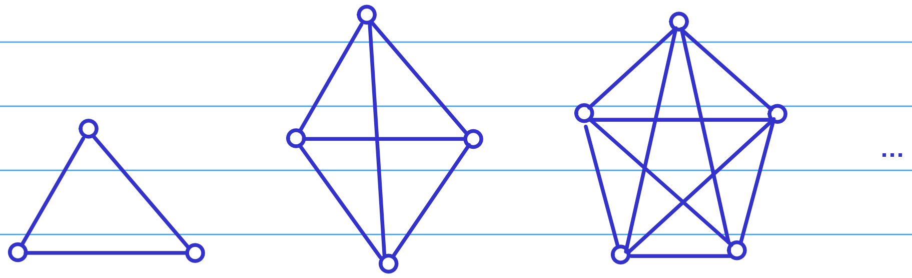


(undirected) graph



directed graph (or digraph)

Complete graph: A graph such that for any two nodes there is an edge between them.



K_3 K_4 K_5 ... K_n

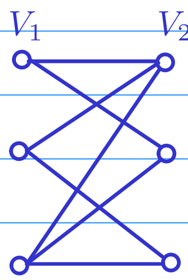
n nodes complete graph

Ex.: a league game

Q: how many edges are there in a complete graph?

=> (n-1) + (n-2) + ... + 1 = n(n-1)/2, or simply (n-1)n/2 by observing the degrees

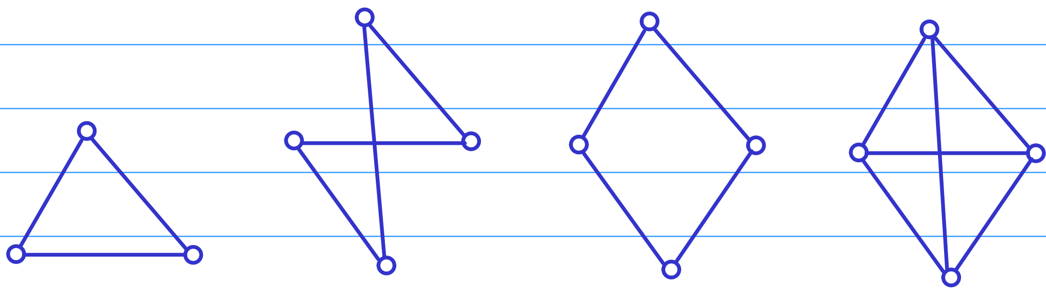
Bipartite graph: a graph $G = (V_1, V_2, E)$ where $V_1 \cap V_2 = \emptyset,\ E \subseteq V_1 \times V_2$



Ex.: V1 = machines, V2 = jobs

Consider two bipartite graphs.

Are the following graphs biparite?



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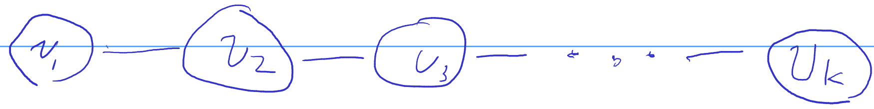
Complete bipartite graph

$K_{m,n}$ where $m = |V_1|, n = |V_2|$ => #edges = mn

Path

$P = v_1, v_2, \ldots, v_k$ is called a path if $(v_1, v_2), (v_2, v_3), \ldots, (v_{k-1}, v_k) \in E$



E.x., a route in a transportation network.

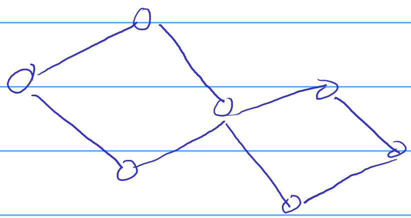
Notice a path can be directed or undirected.

It is said "simple" if $v_i \neq v_j$ if $i \neq j$.

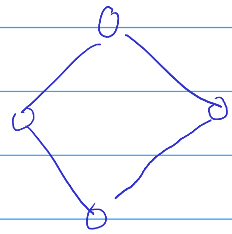
Cycle

is a path such that v_1 = v_k.

It is said "simple" if $v_i \neq v_j$ for any $1 \leq i < j \leq k-1$.



cycle but not simple

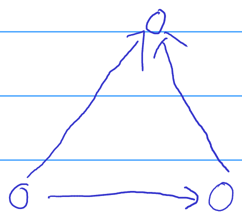


a simple cycle

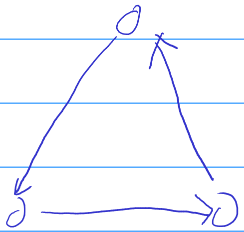
Connectedness

A graph is said "connected" if there exists at least one path for each pair of nodes.

A digraph is said "strongly-connected" if there exists at least one path for each pair of nodes. It is said "weakly-connected" if it is connected when viewed as undirected.



weakly-connected but not strongly



strongly-connected (thus also weakly)

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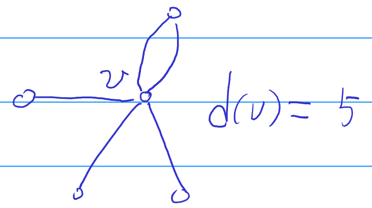
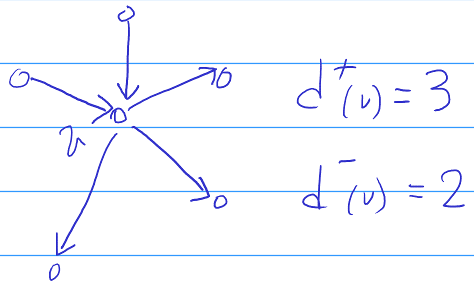
degree

Undirected graph



$d(v) = \#\{(v,w) \in E\}$ i.e., #edges incident to node v

Directed graph



out-degree $d^+(v) = \#\{(v,w) \in E\}$

in-degree $d^-(v) = \#\{(w,v) \in E\}$

Lemma 1. For an undirected graph, $\sum_v d(v) = 2m$.

Proof.

Lemma 2. For a directed graph, $\sum_v d^+(v) = \sum_v d^-(v) = m$.

Proof.

Lemma 3. For an undirected graph, if $d(v) >= 2$ for all nodes v, then there exists at least one cycle in the graph.

Proof. Choose an arbitrary node v_1. Since $d(v_1) >= 2$, there exists a node v_2 such that $(v_1, v_2) \in E$. Again, since $d(v_2) >= 2$, there exists v_3 such that $(v_2, v_3) \in E$. This argument can be repeated to find v_4, ... Since the number of nodes is finite, there must exist some v_k such that $v_k = v_i$ for some $i < k$.
=> $v_i, v_{i+1}, ..., v_k$ is a cycle.

Lemma 4. For a directed graph, if $d^{+}(v) >= 1$ for all nodes v, then there exists at least one directed cycle in the graph. The same is true for $d^{-}(v)$.

Proof. => mini report